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Blow up of solutions to 1-d Euler equations with time-dependent damping

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ABSTRACT

We study the 1-d isentropic Euler equations with time-dependent damping

$$\begin{cases} \partial_t \rho + \partial_x(\rho u) = 0, \\ \partial_t(\rho u) + \partial_x(\rho u^2) + \partial_x p(\rho) = -\dfrac{\mu}{(1+t)^\lambda}\rho u, \\ \rho|_{t=0} = 1 + \varepsilon \rho_0(x), u|_{t=0} = \varepsilon u_0(x). \end{cases}$$

In a previous paper [8], we have proven that, when $\lambda = 1$, $\mu > 2$, the 1-D Euler equations have global existence of small data solutions. However in this paper, we will show that, when the damping, with respect to time, decays faster or equal to $\frac{2}{1+t}$, the C^1 solution of the above system will blow up in finite time. More precisely, when $\lambda = 1$, $0 \le \mu \le 2$ or $\lambda > 1$, $\mu \ge 0$, we will give a finite upper bound for the lifespan. Combining the results in this paper and [8], we see that, when the damping decays with time like $\frac{\mu}{(1+t)^\lambda}$, the critical exponents for λ, μ to separate the global existence and finite-time blow up of small data solutions are $\lambda = 1$, $\mu = 2$.

1. Introduction

This paper deals with the isentropic Euler equations with time-dependent damping in 1 dimension:

$$\begin{cases} \partial_t \rho + \partial_x(\rho u) = 0, \\ \partial_t(\rho u) + \partial_x(\rho u^2) + \partial_x p(\rho) = -\dfrac{\mu}{(1+t)^\lambda}\rho u, \\ \rho|_{t=0} = 1 + \varepsilon \rho_0(x), u|_{t=0} = \varepsilon u_0(x), \end{cases} \tag{1.1}$$

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where $\rho_0(x),\ u_0(x) \in C_0^\infty(\mathbb{R})$, supported in $|x| \le R$ and $\varepsilon > 0$ is sufficient small. Here $\rho(x), u(x)$ and $p(x)$ represent the density, fluid velocity and pressure respectively and λ, μ are two positive constants to describe the decay-rate of the damping concerning time. We assume the fluid is a polytropic gas which means we assume $p(\rho) = \frac{1}{\gamma}\rho^\gamma$, $\gamma > 1$.

As is well known, for 1-d Euler equations with non-decayed damping,

$$\begin{cases} \partial_t\rho + \partial_x(\rho u) = 0, \\ \partial_t(\rho u) + \partial_x(\rho u^2) + \partial_x p(\rho) = -\rho u, \end{cases} \tag{1.2}$$

we have global existence of small data solutions. Many authors have proven this result and given a convergence rate of solutions to system (1.2). See [4,6,7,5,12] etc. and the references therein. However, when the damping is vanishing, the smooth solution of compressible Euler flow will blow up in finite time. For the extensive literature on the blow-up results and the blow-up mechanism, readers can see [1–3,9], and [13,14] and references therein for more details.

It is natural to ask whether the global solution exists when the damping is decayed and what is the critical decay-rate to separate the global existence and the finite-time blow up of solutions with small data. In the paper [8] and this one, we study the case that the damping decays with time as $\frac{\mu}{(1+t)^\lambda}$. We show that the critical exponents for λ, μ are $\lambda = 1$, $\mu = 2$. It means that when $\lambda = 1, \mu > 2$ or $0 \le \lambda < 1$, $\mu > 0$, (1.1) have global existence of small data solutions, while if $\lambda = 1$, $0 \le \mu \le 2$ or $\lambda > 1$, $\mu \ge 0$, the C^1 solutions of (1.1) will blow up in finite time.

Although in [8], we only prove the global existence for $\lambda = 1$, $\mu > 2$. However, by using almost the same weighted energy estimates there and modifying the weight $(1 + t)$ a little bit with $(K + t)$, where K is a suitably large constant, we can show the global existence for $0 \le \lambda < 1$, $\mu > 0$. We will present the details in our later manuscript.

Define the lifespan T_ε be the maximum existing time of C^1 solutions to system (1.1).

We state our main result as follows.

Theorem 1.1. *Define three functions*

$$q^0(r) = \int_r^\infty (x - r)^2 \rho_0(x)dx,$$

$$q^1(r) = 2\int_r^\infty (x - r)u_0(x)dx,$$

$$q^2(r) = 2\int_r^\infty (x - r)(\rho_0 u_0)(x)dx.$$

Suppose $q^0(r) > 0$, $q^1(r)$, $q^2(r) \ge 0$ in some nontrivial interval $[R_0, R]$, where $0 \le R_0 < R$. Then there exists a constant $\varepsilon_0 > 0$ such that for any ε with $0 < \varepsilon \le \varepsilon_0$, the lifespan T_ε of the C^1 solution to system (1.1) is finite and it admits the following upper bound

$$T_\varepsilon \le e^{\frac{C_0}{\varepsilon^2 B_0^2}},$$

where $B_0 = \frac{1}{2}\int_{R_0}^R q^0(r)dr$ and C_0 is a constant depending on $\lambda, \mu, \gamma, R_0, R$.

Remark 1.1. It is easy to find a large quantity of initial data such that $q^0(r) > 0$, $q^1(r)$, $q^2(r) \geq 0$ are satisfied.

The idea of proving **Theorem 1.1** comes from [10] for 3-d compressible Euler equations, but it can not be used directly for the damped Euler equations. We will modify some functions there to make it suitable for our system and prove the finite-time blow up. In general, we will derive a second-order ordinary differential inequality (2.11) with the help of our assumption on λ, μ. Combining this inequality and conditions $q^0 > 0$, $q^1, q^2 \geq 0$, an upper bound of the lifespan T_ε will be obtained.

By the way, using the same technique in proving **Theorem 1 and Theorem2** of [10], we can show that if the initial data is large, the C^1 solution of (1.1) will blow up in finite time for any $\lambda \geq 0$, $\mu \geq 0$. We omit the details.

In this paper, we will use C to denote a generic constant and it may be different from line to line.

In **Section 2**, we will prove **Theorem 1.1**.

2. Proof of Theorem 1.1

In this section, we prove **Theorem 1.1**. We first deal with the case $\gamma = 2$ and later indicate the modification for the general case.

Proof. Case $\gamma = 2$.

Let (ρ, u) be a C^1 solution. By the finite propagation property, we have $\rho - 1$ supported in $B(t) = \{x \mid |x| \leq R + t\}$. We define

$$P(r,t) = \int_{x>r} (x-r)^2(\rho(x,t)-1)dx. \tag{2.1}$$

Using $(1.1)_1$ and integration by parts, we have

$$\begin{aligned}
\partial_t P(r,t) &= \int_{x>r} (x-r)^2 \rho_t dx \\
&= -\int_{x>r} (x-r)^2(\rho u)_x dx \\
&= \int_{x>r} 2(x-r)(\rho u)dx,
\end{aligned}$$

where we have used the fact ρu is supported in $B(t)$ and $(x-r)^2|_{x=r} = 0$. Then $P(r,t)$ is C^2 in t. Differentiating it with time again, using $(1.1)_2$ and integration by parts, we have

$$\begin{aligned}
\partial_t^2 P(r,t) &= \int_{x>r} 2(x-r)(\rho u)_t dx \\
&= \int_{x>r} 2(x-r)\left(-\partial_x(\rho u^2) - \partial_x p - \frac{\mu}{(1+t)^\lambda}\rho u\right) dx \\
&= \int_{x>r} 2\rho u^2 dx + \int_{x>r} 2p dx + \frac{\mu}{(1+t)^\lambda} \int_{x>r} (x-r)^2(\rho u)_x dx \\
&= \int_{x>r} 2\rho u^2 dx + \int_{x>r} 2p dx - \frac{\mu}{(1+t)^\lambda} \partial_t \int_{x>r} (x-r)^2(\rho - 1)dx.
\end{aligned}$$

Hence we have

$$\partial_t^2 P(r,t) + \frac{\mu}{(1+t)^\lambda}\partial_t P(r,t) = \int\limits_{x>r} 2\rho u^2 dx + \int\limits_{x>r} 2p dx \\ \geq 0.$$

Our initial data assumption on $P(r,t)$ is

$$P(r,t)|_{t=0} = \varepsilon q^0(r) > 0, \qquad \partial_t P(r,t)|_{t=0} = \varepsilon q^1(r) + \varepsilon^2 q^2(r) \geq 0.$$

By integrating the above differential inequality, we have $\partial_t P(r,t) \geq 0$ and $P(r,t) > 0$.

Now we come to estimate a lower bound for $P(r,t)$. Rewrite $\partial_t^2 P(r,t)$ as follows.

$$\begin{aligned}
\partial_t^2 P(r,t) &= \int\limits_{x>r} 2(x-r)(\rho u)_t dx \\
&= \int\limits_{x>r} 2(x-r)\left(-\partial_x(\rho u^2) - \partial_x p - \frac{\mu}{(1+t)^\lambda}\rho u\right) dx \\
&= \int\limits_{x>r} 2(x-r)\left(-\partial_x(\rho u^2) - \partial_x (p-\overline{p}) - \frac{\mu}{(1+t)^\lambda}\rho u\right) dx \\
&= \int\limits_{x>r} 2\rho u^2 dx + \int\limits_{x>r} 2(p-\overline{p}) dx + \frac{\mu}{(1+t)^\lambda}\int\limits_{x>r} (x-r)^2(\rho u)_x dx \\
&\geq \int\limits_{x>r} 2(p-\overline{p}) dx - \frac{\mu}{(1+t)^\lambda}\partial_t \int\limits_{x>r} (x-r)^2(\rho-1) dx,
\end{aligned}$$

where $\overline{p} = p(1) = \frac{1}{\gamma}$. From the definition of $P(r,t)$, we have

$$\partial_r^2 P(r,t) = \int\limits_{x>r} 2(\rho-1) dx.$$

Then we have

$$\begin{aligned}
\partial_t^2 P(r,t) - \partial_r^2 P(r,t) + \frac{\mu}{(1+t)^\lambda}\partial_t P(r,t) &\geq \int\limits_{x>r} 2(p-\overline{p}) dx - \int\limits_{x>r} 2(\rho-1) dx \\
&= \frac{2}{\gamma}\int\limits_{x>r} \left[(\rho^\gamma - 1) - \gamma(\rho-1)\right] dx \\
&\triangleq G(r,t).
\end{aligned} \tag{2.2}$$

When $\gamma = 2$,

$$G(r,t) = \int\limits_{x>r} (\rho-1)^2 dx \geq 0.$$

When $\lambda = 1$, $0 \leq \mu \leq 2$ or $1 < \lambda$, $0 \leq \mu$, due to the nonnegativity of $\partial_t P$, we have

$$\begin{aligned}
&\partial_t^2 P(r,t) - \partial_r^2 P(r,t) + \frac{2}{1+t}\partial_t P(r,t) \\
&\geq \partial_t^2 P(r,t) - \partial_r^2 P(r,t) + \frac{\mu}{(1+t)^\lambda}\partial_t P(r,t) \\
&\geq G(r,t) \qquad (2.3)
\end{aligned}$$

when $t > t_0 \triangleq \max\{(\frac{\mu}{2})^{\frac{1}{\lambda-1}} - 1, 0\}$. For simplicity, we just set $t_0 = 0$. Otherwise we can shift the time from t to $\tilde{t} = t + t_0$. All the following estimates and inequalities are effective.

Define $W(r,t) = (1+t)P(r,t)$. From the above inequality, one gets

$$\partial_t^2 W(r,t) - \partial_r^2 W(r,t) \geq (1+t)G(r,t). \qquad (2.4)$$

We see

$$W(r,0) = \varepsilon q^0(r), \qquad (\partial_t W)(r,0) = \varepsilon(q^0(r) + q^1(r)) + \varepsilon^2 q^2(r).$$

Inversion of 1-d d'Alembertian operator gives (for $r > R_0 + t$)

$$\begin{aligned}
W(r,t) &= W^0(r,t) + \frac{1}{2}\int_0^t \int_{r-(t-\tau)}^{r+t-\tau} \Box W(y,\tau)dyd\tau \\
&\geq W^0(r,t) + \frac{1}{2}\int_0^t \int_{r-(t-\tau)}^{r+t-\tau} (1+\tau)G(y,\tau)dyd\tau,
\end{aligned}$$

where

$$W^0(r,t) = \frac{\varepsilon}{2}\left\{ q^0(r+t) + q^0(r-t) + \int_{r-t}^{r+t} (q^0(y) + q^1(y) + \varepsilon q^2(y))dy \right\}.$$

Now define

$$F(t) = \int_0^t (t-\tau) \int_{R_0+\tau}^{R+\tau} r^{-1} W(r,\tau)drd\tau.$$

We see that

$$\begin{aligned}
F''(t) &= \int_{R_0+t}^{R+t} r^{-1} W(r,t)dr \\
&\geq \int_{R_0+t}^{R+t} r^{-1} W^0(r,t)dr \\
&\quad + \frac{1}{2}\int_{R_0+t}^{R+t} r^{-1} \int_0^t \int_{r-(t-\tau)}^{r+t-\tau} (1+\tau)G(y,\tau)dyd\tau dr \\
&= J_1 + J_2. \qquad (2.5)
\end{aligned}$$

From our assumption on $q^0(r)$, we have

$$\begin{aligned}
J_1 &\geq \varepsilon \frac{1}{2} \int\limits_{R_0+t}^{R+t} r^{-1} q^0(r-t)dr \\
&\geq \varepsilon (R+t)^{-1} \frac{1}{2} \int\limits_{R_0+t}^{R+t} q^0(r-t)dr \\
&\geq \varepsilon (R+t)^{-1} B_0 \\
&> 0,
\end{aligned} \tag{2.6}$$

where $B_0 = \frac{1}{2}\int_{R_0}^{R} q^0(r)dr$.

Exchanging the order of integration in J_2 and remembering that $G(y,\tau)$ is supported in $\{y \mid |y| \leq R+\tau\}$, we have

$$J_2 \geq \frac{1}{2} \int\limits_0^t \int\limits_{R_0+\tau}^{R+\tau} (1+\tau)G(y,\tau) \int\limits_{\max[R_0+t, y-(t-\tau)]}^{y+t-\tau} r^{-1} dr dy d\tau.$$

If we set $t \geq t_1 \triangleq \max\{\frac{R-R_0}{2}, t_0\}$, by direct computation, we have

$$\int\limits_{\max[R_0+t, y-(t-\tau)]}^{y+t-\tau} r^{-1} dr \geq C(R+t)^{-2}(t-\tau)(y-R_0-\tau)^2.$$

Since $G(y,\tau) \geq 0$, we have

$$J_2 \geq C(R+t)^{-2} \int\limits_0^t \int\limits_{R_0+\tau}^{R+\tau} (t-\tau)(y-R_0-\tau)^2(1+\tau)G(y,\tau)dyd\tau, \tag{2.7}$$

when $t > t_1$. We know that $G(y,\tau)$ is supported in $\{y \mid |y| \leq R+\tau\}$ and

$$G(y,\tau) = \partial_y^2 \int\limits_{x>y} (x-y)^2(\rho-1)^2 dx.$$

Using integration by parts in (2.7), we have

$$\begin{aligned}
J_2 &\geq C(R+t)^{-2} \int\limits_0^t \int\limits_{R_0+\tau}^{R+\tau} \int\limits_{x>y} (t-\tau)(1+\tau)(x-y)^2(\rho-1)^2 dx dy d\tau \\
&= C(R+t)^{-2} J_3.
\end{aligned} \tag{2.8}$$

Recall that

$$F(t) = \int\limits_0^t (t-\tau) \int\limits_{R_0+\tau}^{R+\tau} y^{-1} W(y,\tau) dy d\tau$$

$$= \int_0^t \int_{R_0+\tau}^{R+\tau} (t-\tau)y^{-1}(1+\tau) \int_{x>y} (x-y)^2(\rho-1)dxdyd\tau$$

$$= \int_0^t \int_{R_0+\tau}^{R+\tau} \int_{x>y} (t-\tau)y^{-1}(1+\tau)(x-y)^2(\rho-1)dxdyd\tau.$$

Using Hölder inequality and the finite propagation of ρ, we have

$$\begin{aligned} F^2(t) &\le J_3 \int_0^t \int_{R_0+\tau}^{R+\tau} \int_y^{R+\tau} (t-\tau)(1+\tau)y^{-2}(x-y)^2 dxdyd\tau \\ &= J_3 J_4. \end{aligned} \tag{2.9}$$

We compute J_4 as follows

$$\begin{aligned} J_4 &= \frac{1}{3}\int_0^t (t-\tau)(1+\tau) \int_{R_0+\tau}^{R+\tau} y^{-2}(R+\tau-y)^3 dyd\tau \\ &\le \frac{(R-R_0)^3}{3} \int_0^t (t-\tau)(1+\tau) \int_{R_0+\tau}^{R+\tau} y^{-2} dyd\tau \\ &\le C\int_0^t (t-\tau)(1+\tau)\frac{1}{(R_0+\tau)^2} d\tau \\ &\le C(R+t)\ln(R+t). \end{aligned} \tag{2.10}$$

Combining (2.5)–(2.10), we get

$$F''(t) \ge C[(R+t)^3 \ln(R+t)]^{-1} F^2(t), \qquad t \ge t_1. \tag{2.11}$$

From (2.5), (2.6) and the fact $J_2 \ge 0$, $F'(0) = F(0) = 0$, we have

$$F''(t) \ge \varepsilon B_0 (R+t)^{-1}, \qquad t \ge t_1, \tag{2.12}$$

$$F'(t) \ge \varepsilon B_0 \ln\left(\frac{R+t}{R}\right), \qquad t \ge t_1, \tag{2.13}$$

$$F(t) \ge C\varepsilon B_0 (R+t)\ln\left(\frac{R+t}{R}\right), \qquad t \ge t_2, \tag{2.14}$$

where $t_2 = \max\{t_1, R(e^2-1)\}$.

Actually from (2.11), (2.12), (2.13) and (2.14), we can deduce the blow up as in [11]. However for completion of our paper, we sketch the proof in the following. For simplicity, we set $R = 1$.

Inserting (2.14) into (2.11), one obtains the improvement for $F''(t)$

$$F''(t) \ge C\varepsilon^2 B_0^2 (1+t)^{-1} \ln(1+t), \qquad t \ge t_2. \tag{2.15}$$

Integrating (2.15) twice, we have

$$F(t) \ge C\varepsilon^2 B_0^2 (1+t) \left[\ln(1+t)\right]^2, \qquad t \ge t_2. \tag{2.16}$$

Inserting (2.16) into (2.11), we have

$$F''(t) \geq C\varepsilon^2 B_0^2 (1+t)^{-2} \ln(1+t) F(t), \qquad t \geq t_2. \tag{2.17}$$

Multiplying both sides of (2.17) by $F'(t)(\geq 0)$, we have

$$[(F'(t))^2]' \geq C\varepsilon^2 B_0^2 (1+t)^{-2} \ln(1+t)[(F(t))^2]'. \tag{2.18}$$

Integrating (2.18) from some $t_3 \geq t_2$ to t, we have

$$(F'(t))^2 \geq (F'(t_3))^2 + C\varepsilon^2 B_0^2 \int_{t_3}^{t} (1+\tau)^{-2} \ln(1+\tau)[F^2(\tau)]' d\tau.$$

When ε is small, first we can choose $t_3 \geq t_2$ such that

$$C\varepsilon^2 B_0^2 \ln(1+t_3) = 1,$$

which implies

$$t_3 = e^{\frac{C}{\varepsilon^2 B_0^2}} - 1. \tag{2.19}$$

Then we have

$$(F'(t))^2 \geq (F'(t_3))^2 + \frac{1}{\ln(1+t_3)} \int_{t_3}^{t} (1+\tau)^{-2} \ln(1+\tau)[F^2(\tau)]' d\tau.$$

Using integration by parts, we have

$$\begin{aligned}
(F'(t))^2 \geq{}& (F'(t_3))^2 + \frac{(1+t)^{-2}\ln(1+t)}{\ln(1+t_3)} F^2(t) \\
&- (1+t_3)^{-2} F^2(t_3) - \frac{1}{\ln(1+t_3)} \int_{t_3}^{t} F^2(\tau) \frac{1-2\ln(1+\tau)}{(1+\tau)^3} d\tau \\
\geq{}& \frac{(1+t)^{-2}\ln(1+t)}{\ln(1+t_3)} F^2(t) \\
&+ (F'(t_3))^2 - (1+t_3)^{-2} F^2(t_3).
\end{aligned} \tag{2.20}$$

Here we have used the fact when $t \geq t_3$, we have $1 - 2\ln(1+t) \leq 0$. Noting that $F'(t)$ is increasing and $F(0) = 0$, we have

$$F(t_3) \leq t_3 F'(t_3). \tag{2.21}$$

Substituting (2.21) into (2.20), we have

$$F'(t) \geq C(1+t)^{-1}(\ln(1+t))^{\frac{1}{2}} F(t) \qquad t \geq t_3.$$

Integrating this from t_3 to t, one obtains

$$\ln \frac{F(t)}{F(t_3)} = \frac{2}{3}C(\ln(1+t))^{3/2} - \frac{2}{3}C(\ln(1+t_3))^{3/2}.$$

Choosing $t_4 = 2t_3^2$ and noting (2.16), we have

$$\begin{aligned} F(t) &\geq F(t_3)e^{C(\ln(1+t))^{3/2}} \\ &\geq C\varepsilon^2 B_0^2(1+t_3)\left[\ln(1+t_3)\right]^2 e^{C(\ln(1+t))^{3/2}} \\ &\geq C\varepsilon^2 B_0^2(1+t)^8, \end{aligned} \tag{2.22}$$

when $t \geq t_4$.

Inserting this into (2.11), we get

$$F''(t) \geq C\varepsilon B_0 F(t)^{3/2}, \qquad t \geq t_4.$$

Integrating, as before, the above differential inequality, we get

$$(F'(t))^2 \geq C\varepsilon B_0\left((F(t))^{5/2} - (F(t_4))^{5/2}\right), \qquad t \geq t_4.$$

On the other hand, due to the nonnegative of $F'(t)$ and $F''(t)$, we have

$$F(t) \geq F(t_4) + F'(t_4)(t-t_4) \geq F'(t_4)(t-t_4) \geq F(t_4)\frac{t-t_4}{t_4}.$$

Then choosing $t_5 = 3t_4$, we get

$$F'(t)) \geq C\sqrt{\varepsilon B_0}(F(t))^{5/4}, \qquad t \geq t_5.$$

If the lifespan $T_\varepsilon > t_5$, integrating the above inequality from t_5 to T_ε gives

$$(F(t_5))^{-1/4} - (F(T_\varepsilon))^{-1/4} \geq C\sqrt{\varepsilon B_0}T_\varepsilon. \tag{2.23}$$

Noting that we have chosen $t_5 = 6t_3^2$ and the inequalities (2.16) and (2.19). Then we have

$$F(t_5) \geq C\varepsilon^2 B_0^2 e^{\frac{C}{\varepsilon^2 B_0^2}}.$$

Combined with (2.23), we can deduce that

$$T_\varepsilon \leq C(\varepsilon B_0)^{-1}e^{-\frac{C}{\varepsilon^2 B_0^2}}.$$

However this contradicts with $T_\varepsilon > 6t_3^2 = 6\big(e^{\frac{C}{\varepsilon^2 B_0^2}} - 1\big)^2$ when ε is small.

So $T_\varepsilon \leq t_5$, then by our choice of t_5 and (2.19)

$$T_\varepsilon \leq 6t_3^2 \leq e^{\frac{C}{\varepsilon^2 B_0^2}}.$$

Case $1 < \gamma < 3$ and $\gamma \neq 2$.

For the general case, we need to adjust the function $G(r,t)$ in (2.2). Using Taylor's theorem, we have

$$(\rho^\gamma - 1) - \gamma(\rho - 1) = \gamma(\gamma-1)\int_1^\rho \tau^{\gamma-2}(\rho-\tau)d\tau.$$

It is easy to see that

$$\int_1^\rho \tau^{\gamma-2}(\rho-\tau)d\tau \geq C(\gamma)\varphi_\gamma(\rho),$$

where $C(\gamma)$ is a constant and φ_γ is given by

$$\varphi_\gamma(\rho) = \begin{cases} (1-\rho)^\gamma, & 0 < \rho < \frac{1}{2}, \\ (\rho-1)^2, & \frac{1}{2} \leq \rho \leq 2, \\ (\rho-1)^\gamma, & \rho > 2. \end{cases}$$

Then

$$G(r,t) \geq C(\gamma) \int_{x>r} \varphi_\gamma(\rho)dx.$$

Young inequalities will be used in (2.9). We can still get similar inequalities as (2.11), (2.12), (2.13) and (2.14) to prove the finite-time blow up, although the upper bound for the lifespan will be a little different. We omit the details.

This finishes the proof of **Theorem 1.1**. □

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